Non-equilibrium fluctuations in chiral fluid dynamics at the QCD phase transition

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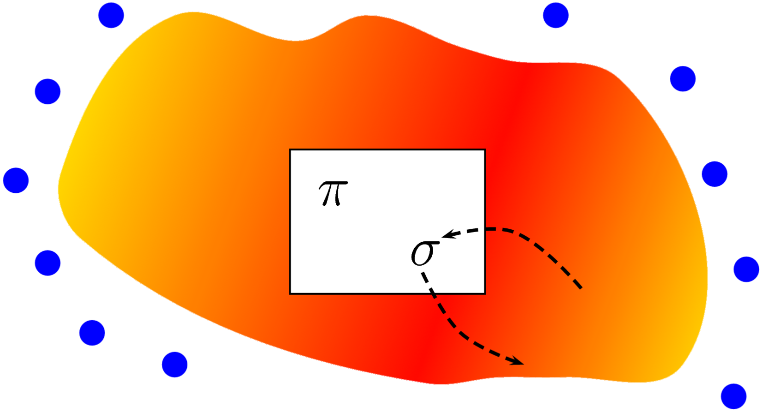
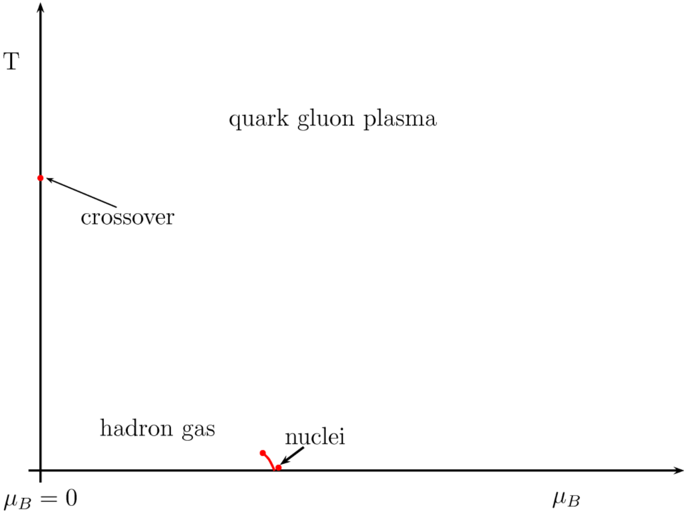
Chiral fluid dynamics

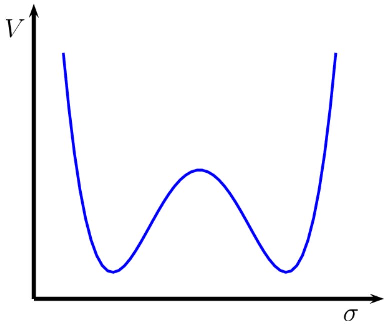


QCD phase transition

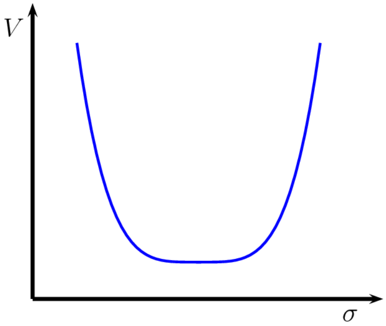


Phase transitions

first order phase transition



critical point

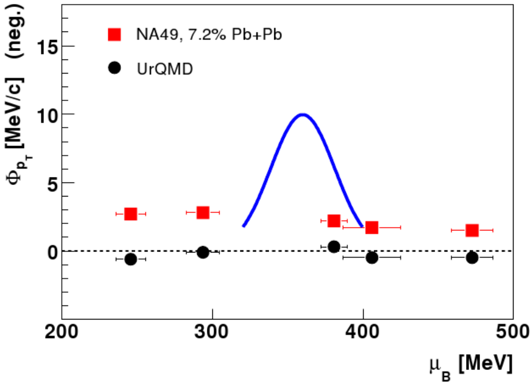


I. N. Mishustin, Phys. Rev. Lett. **82** (1999)

J. Randrup, Phys. Rev. Lett. **92** (2004)

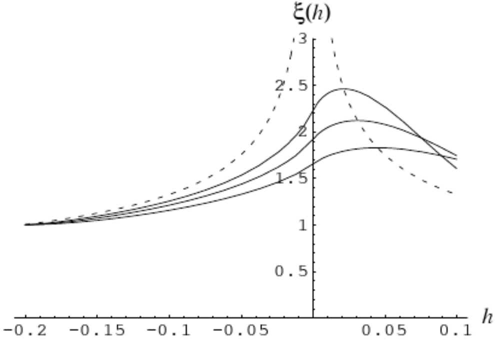
Critical phenomena

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$



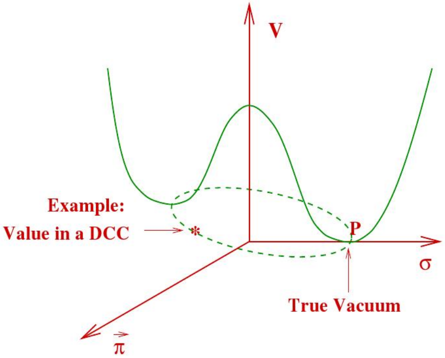
(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. D **60** (1999), NA49 collaboration J. Phys. G **35** (2008))

The critical point in dynamic systems



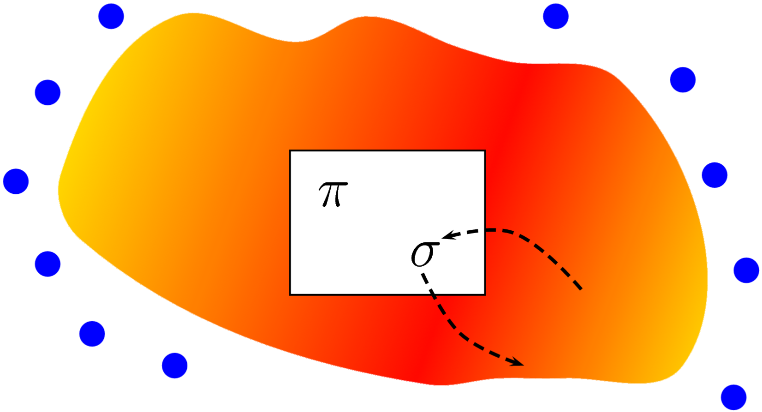
(B. Berdnikov and K. Rajagopal, Phys. Rev. D **61** (2000))

Disoriented chiral condensates



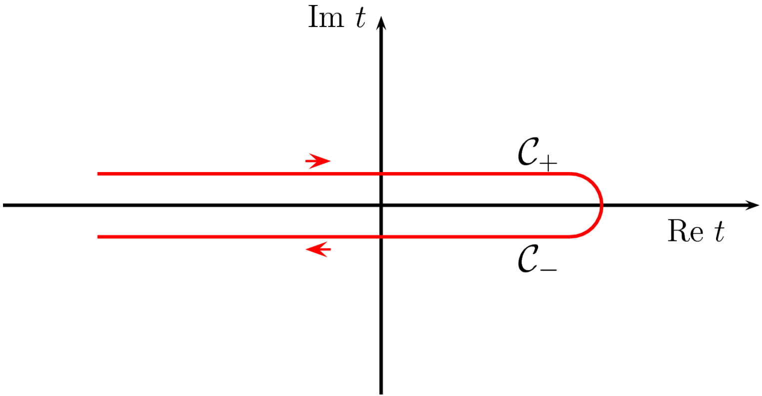
(K. Rajagopal and F. Wilczek, Nucl. Phys. B **404** (1993)
Z. Xu and C. Greiner, Phys. Rev. D **62** (2000)
D. H. Rischke, Phys. Rev. C **58** (1998))

Chiral fluid dynamics

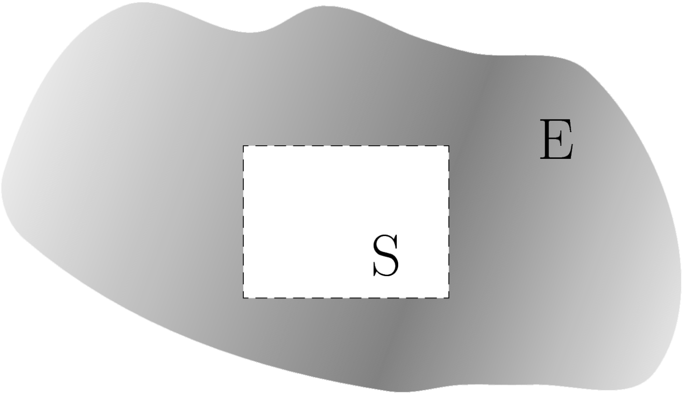


I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999)
K. Paech, H. Stoecker and A. Dumitru, Phys. Rev. C **68** (2003)

Real time formalism - closed time path formalism



Influence functional method



$$S[x, q] = S_{\mathrm{S}}[x] + S_{\mathrm{E}}[q] + S_{\mathrm{int}}[x, q]$$

Influence functional method

reduced density matrix for the system variables

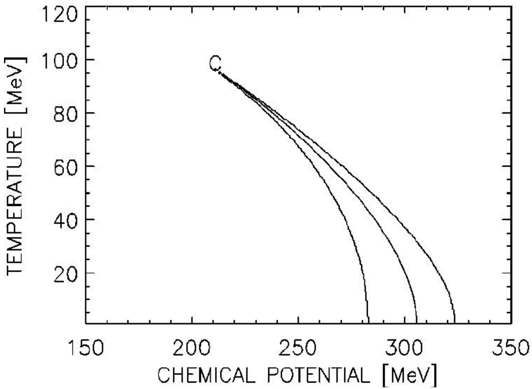
$$\rho^{\mathrm{S}}(x, x', t) = \int \mathrm{d}x_i \mathrm{d}x_i' \rho_i^{\mathrm{S}}(x_i, x_i', t_i) \times$$
$$\times \int_{x_i}^{x} \mathcal{D}x \int_{x_i'}^{x'} \mathcal{D}x' \times$$
$$\times \exp[i(S_{\mathrm{S}}[x] - S_{\mathrm{S}}[x'] + S_{\mathrm{IF}}[x, x'])]$$

Linear sigma model with constituent quarks

$$\begin{aligned}\mathcal{L} = & \bar{q}(i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\tau\vec{\pi}))q \\ & + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 \\ & - U(\sigma,\vec{\pi})\end{aligned}$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - h_q\sigma - U_0$$

Phase diagram of the linear sigma model with constituent quarks



(O. Scavenius, A. Mocsy, I. N. Mishustin and D. H. Rischke, Phys. Rev. C **64** (2001))

Classical equation of motion for the sigma field

expand $\underbrace{S_{\mathrm{S}}[\sigma] - S_{\mathrm{S}}[\sigma']}_{\to \text{ ordinary eom}} + S_{\mathrm{IF}}[\sigma, \sigma']$

$\downarrow$

$$S_{\mathrm{IF}} = \int \mathrm{d}^4 x D(x) \Delta\sigma(x)$$

$$+ \frac{i}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x) \mathcal{I}(x, y) \Delta\sigma(y)$$

Classical equation of motion for the sigma field

$$\exp[-\tfrac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Delta\sigma(x)\mathcal{I}(x,y)\Delta\sigma(y)]$$

$$\equiv \int \mathcal{D}\xi P[\xi] \exp[i \int \mathrm{d}^4x \xi(x)\Delta\sigma(x)]$$

↑

Gaussian measure with $\langle\xi\rangle = 0$

and $\langle\xi(t)\xi(t')\rangle = \mathcal{I}^{-1}(t,\mathbf{x};t',\mathbf{y})$

Classical equation of motion for the sigma field

fluctuations

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\langle \bar{q} q \rangle_\sigma = \xi$$

interaction between the field and the heat bath

Chiral condensate - lowest order

$$\langle \bar{q}q \rangle_{\sigma}^{(0)} = S_{++}(0) = 2 d_q m_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{n_{\mathrm{F}}(p)}{E}$$

$$m_q = g\langle \sigma \rangle \quad , \quad E = \sqrt{p^2 + g^2\sigma^2}$$

equation of motion for the sigma field:

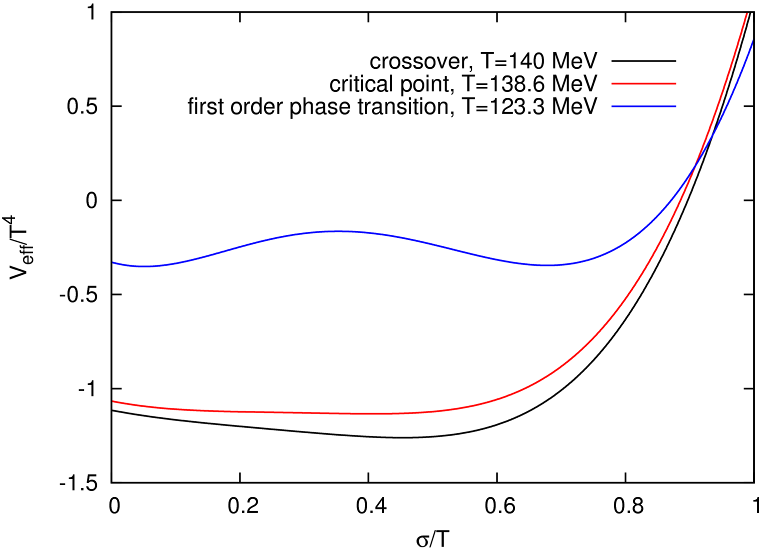
$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\langle \bar{q}q \rangle_0 = 0$$

Effective potential - lowest order

$$\mathcal{Z} = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi}\, \exp\left[\int_0^{1/T} \mathrm{d}(it) \int_V \mathrm{d}^3x\, \mathcal{L}\right]$$

$$V_{\text{eff}} = -\frac{T}{V}\log \mathcal{Z} = -\,2d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \log\left(1 + \mathrm{e}^{-E/T}\right) + U\left(\sigma, \vec{\pi}\right)$$

Effective potential - lowest order

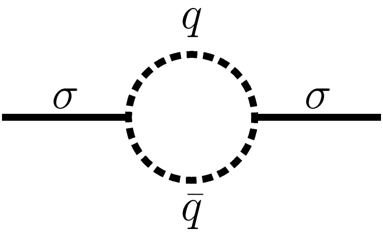


Chiral condensate - first order

$$\langle \bar{q}q \rangle_\sigma^{(1)} = ig \int \mathrm{d}y^4 \mathrm{Tr}\big(S_>(x-y)S_<(y-x) - S_<(x-y)S_>(y-x)\big)\sigma(y)$$

Damping term η and noise ξ

for $\mathbf{k} = 0$



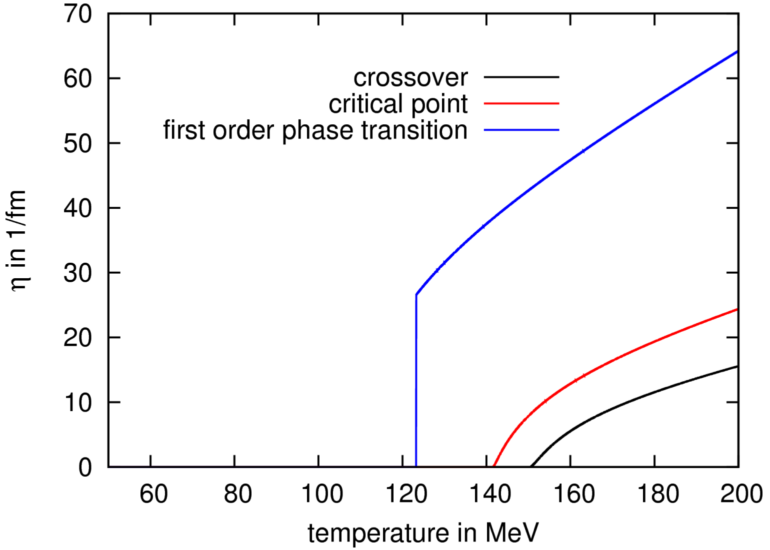
$$\eta = g^2 \frac{d_q}{\pi} (1 - 2 n_{\mathrm{F}} (\frac{m_\sigma}{2})) \frac{(\frac{m_\sigma^2}{4} - m_q^2)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle \xi(t) \xi(t') \rangle_\xi = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth(\frac{m_\sigma}{2T})$$

Equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \langle \bar{q} q \rangle_\sigma^{(0)} + \eta \partial_t \sigma = \xi$$

Damping term η



Choice of the damping coefficients

$$\eta_1 = 2.2/\mathrm{fm} \quad \text{and} \quad \eta_2 = 20/\mathrm{fm}$$

$$\begin{aligned} \langle \xi(t) \rangle &= 0 \\ \langle \xi(t)\xi(t') \rangle &= \frac{2T}{V}\eta_{1/2}\delta(t-t') \end{aligned}$$

(T. S. Biro and C. Greiner, Phys. Rev. Lett. **79** (1997))

Stochastic source term

$$\partial_\mu T^{\mu\nu} = S_\nu$$

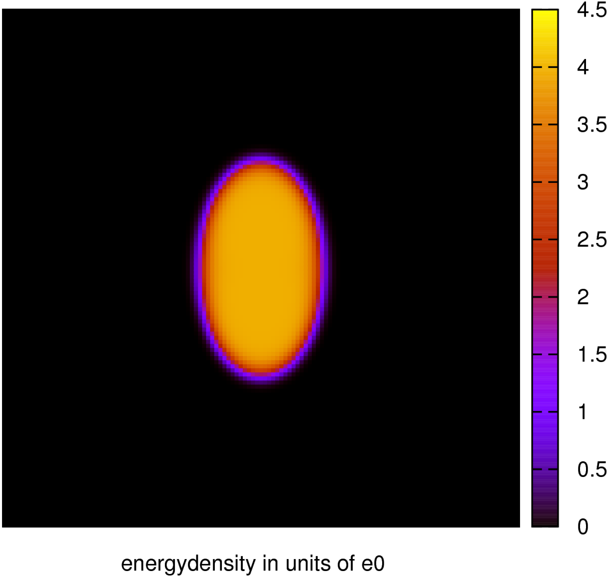
$$\begin{aligned} S^\nu = -\partial_\mu T^{\mu\nu}_{\text{field}} &= -\left(\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma}\right)\partial^\nu \sigma \\ &= -\left(-g\langle \bar{q}q\rangle^{(0)}_\sigma - \eta \partial_t \sigma + \xi\right)\partial^\nu \sigma \end{aligned}$$

The equation of state

$$e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)$$

$$p(\sigma, T) = -V_{\text{eff}}(\sigma, T) + U(\sigma)$$

Initial conditions



Intensity of sigma fluctuations

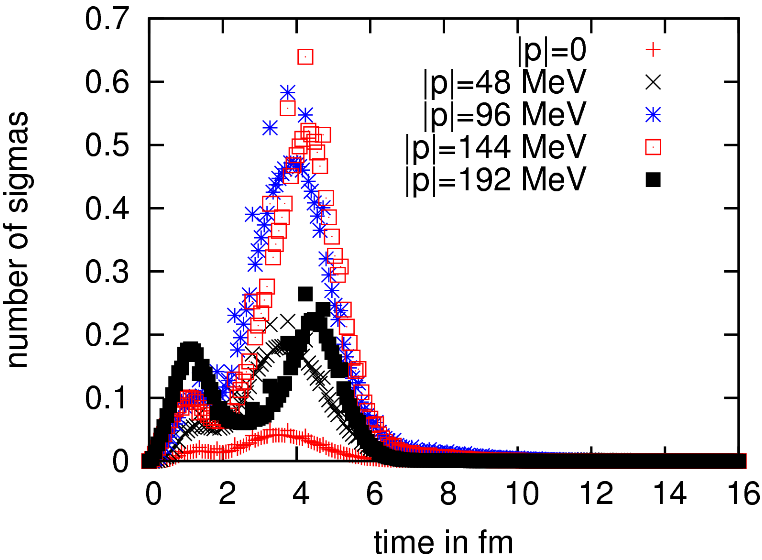
$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \frac{1}{(2\pi)^3 2\omega_k}(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)$$

$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

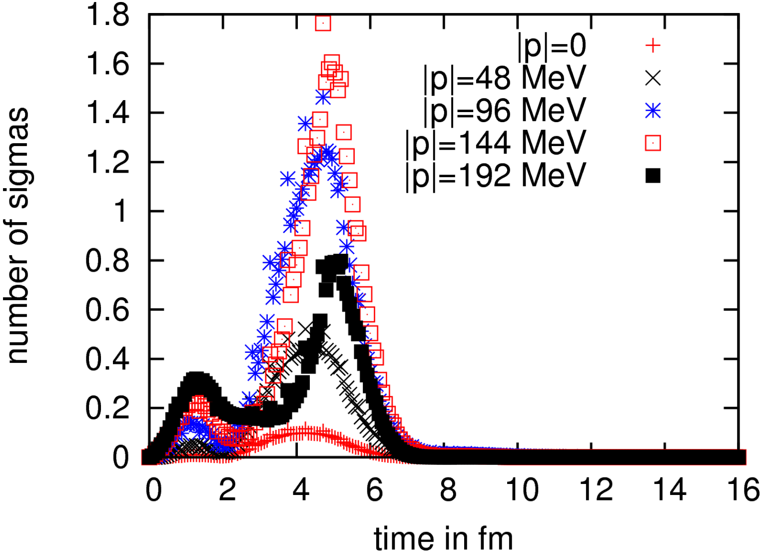
$$m_\sigma = \sqrt{\frac{\partial^2 V_{\mathrm{eff}}}{\partial\sigma^2}|_{\sigma=\sigma_{\mathrm{eq}}}}$$

$$\eta_1 = 2.2/\,\mathrm{fm}$$

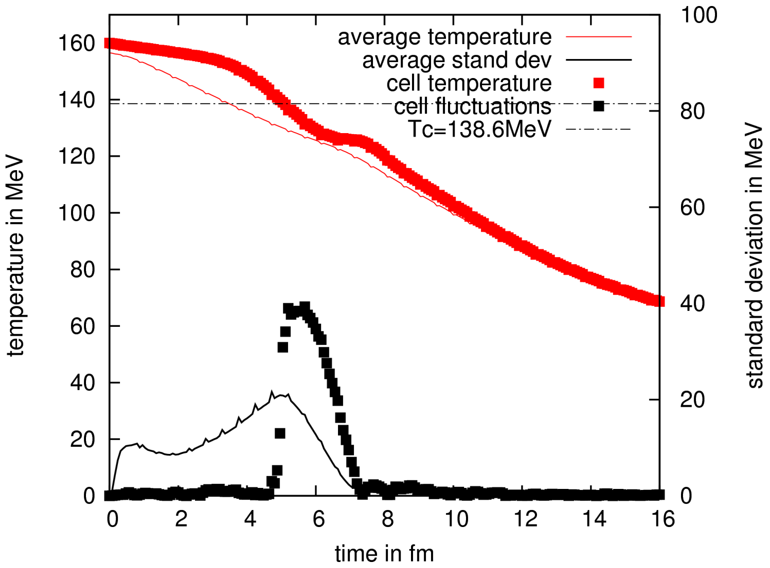
Intensity of sigma fluctuations - crossover



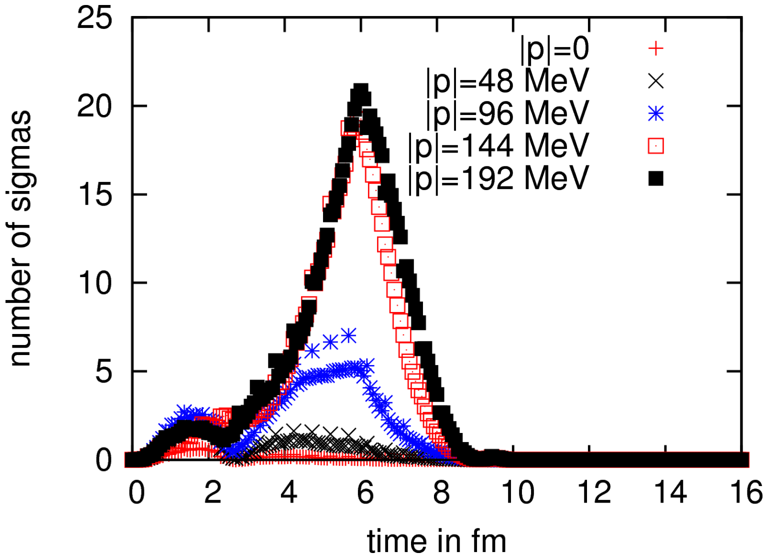
Intensity of sigma fluctuations - critical point



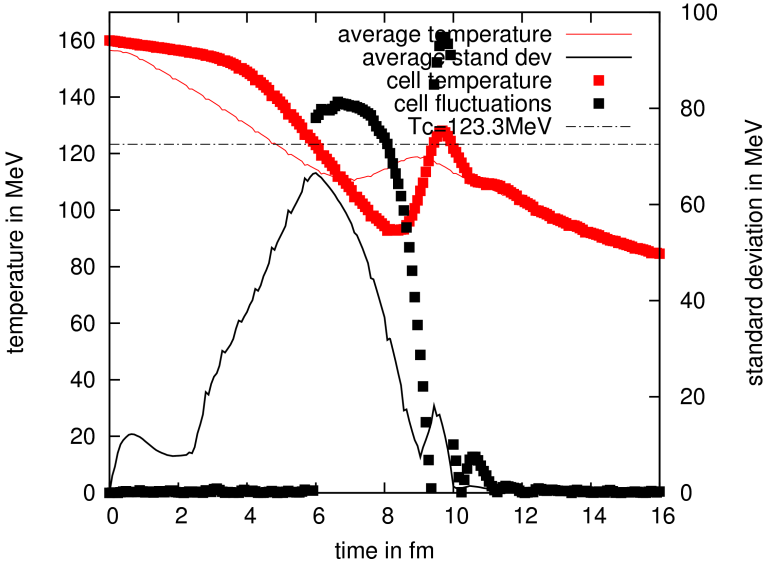
Fluctuations - critical point



Intensity of sigma fluctuations - first order phase transition



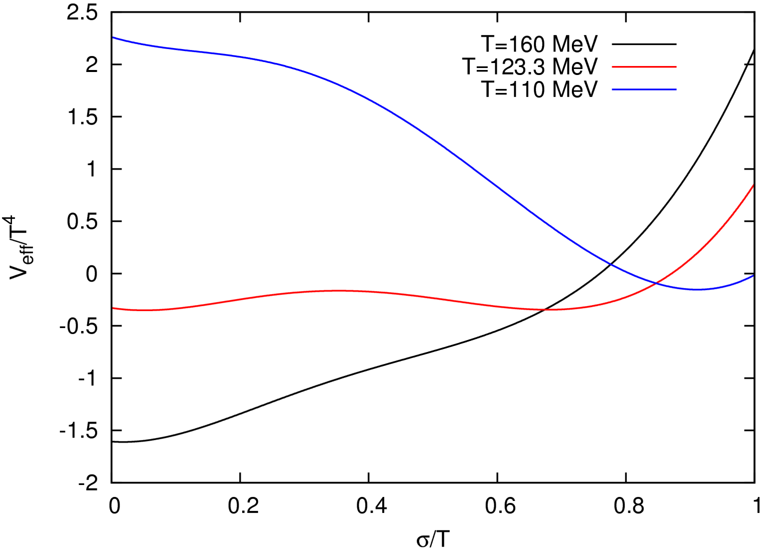
Fluctuations - first order phase transition



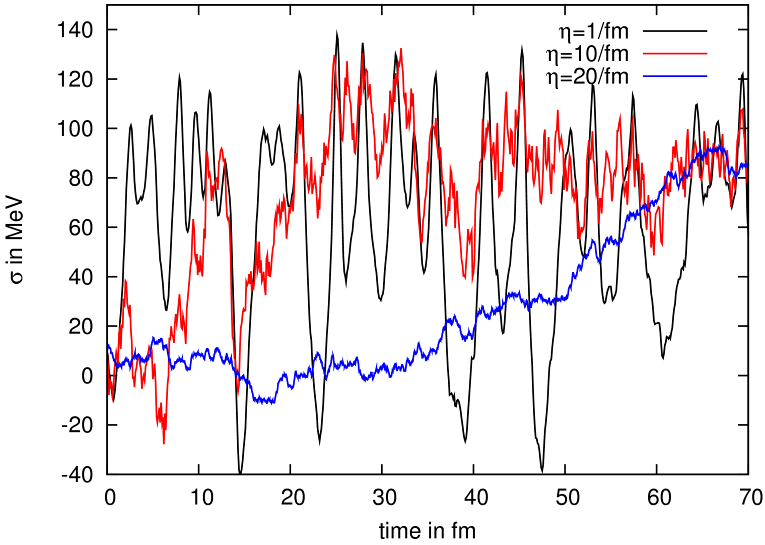
Larger damping?

$$\eta_2 = 20/\mathrm{fm}$$

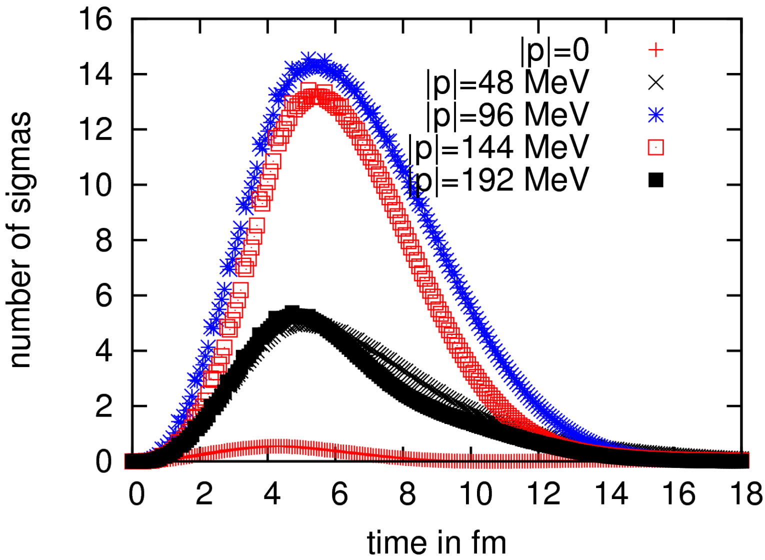
Effective potential



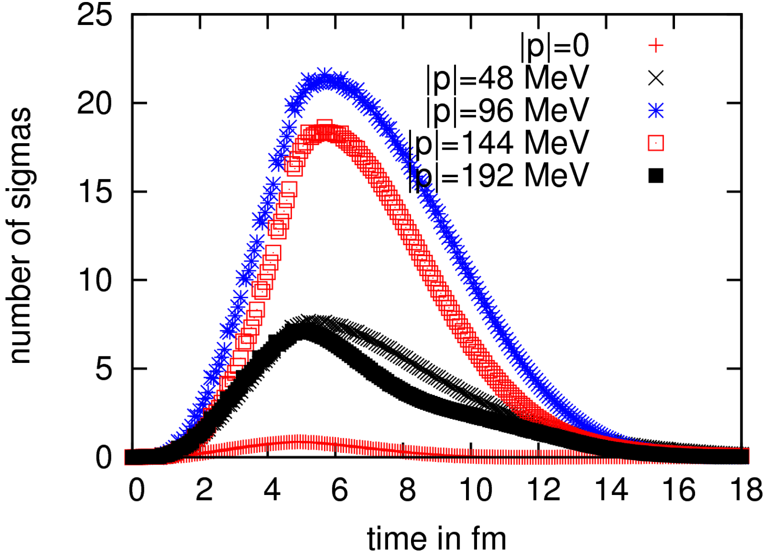
Relaxation of the sigma field



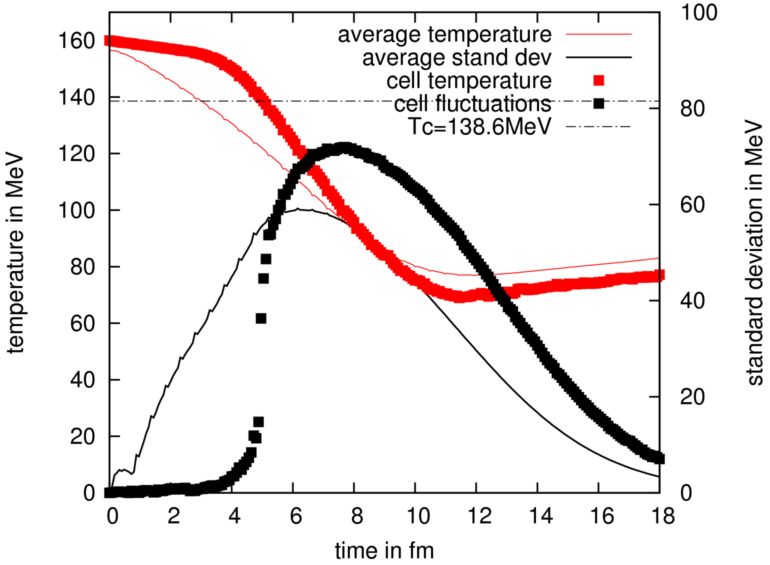
Intensity of sigma fluctuations - crossover



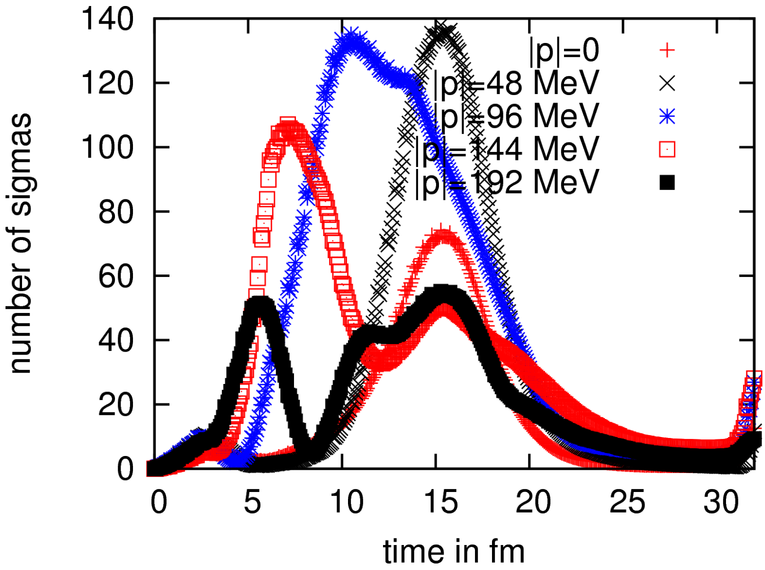
Intensity of sigma fluctuations - critical point



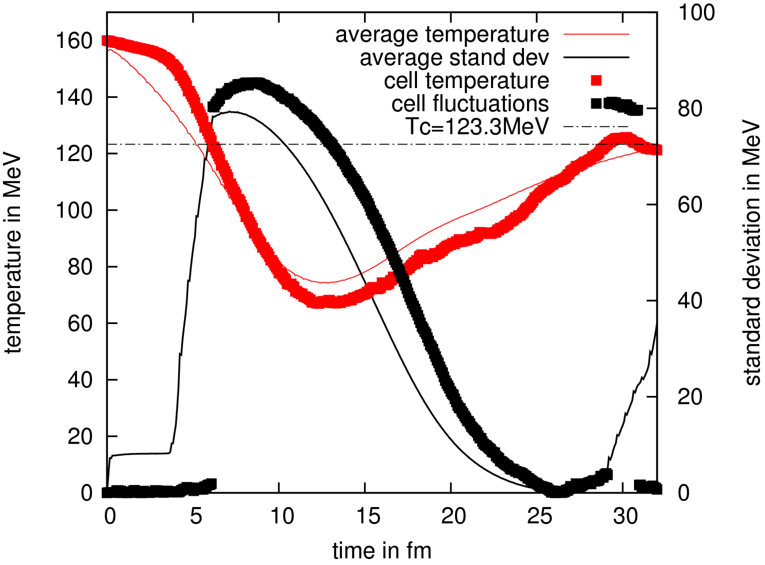
Intensity of sigma fluctuations - critical point



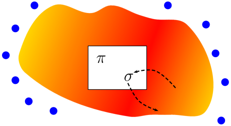
Intensity of sigma fluctuations - first order phase transition

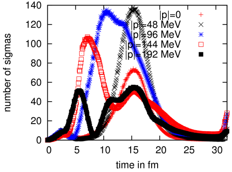


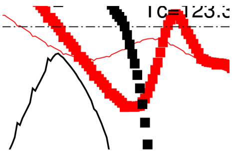
Fluctuations - first order phase transition

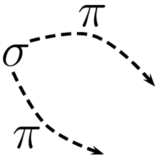


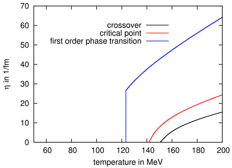
Summary & outlook

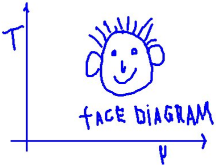












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